Comment on "Optimum Placement of Controls for Static Deformations of Space Structures"

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IN an interesting paper,[1] Professor Haftka describes a method for optimum control of static deformations of space structures. However, it must be emphasized that not only are the deformations of the space structures static but, appropriately, the means of control are also static. Having this in mind, it is not apparent to this author why Prof. Haftka had to involve the mass of the structure in his equations. In an approach somewhat different to that given in the paper,[1] it will be shown here that one can develop adequate equations for the static control of a space structure without involving the mass of the structure.

It will be assumed that the free space structure is represented by its stiffness matrix $K(n\times n)$, usually obtained by using finite element methods. This matrix is needed to calculate the deformations of the structure constrained in a statically definite way. The distortion from the desired shape of the structure is represented by the deformation vector $\psi(n\times 1)$. Thermal or force controls are applied on the structure to minimize the distortion. The rigid body displacements of the structure are characterized by the matrix $R(n\times r)$. (For space structures, $r=6$.) For convenience, the matrix R can be orthonormalized to obtain

$$R^T R = I \tag{1}$$

The vector ψ is assumed to represent only shape distortion. Hence, it is orthogonal to R,

$$R^t \psi = 0 \tag{2}$$

The vector $\Delta T(m\times 1)$ represents the temperature of m heating control points on the structure. By restraining the structure in a statically definite way, one can calculate the deformations caused by applying a separate unit temperature at any one of the control points. The result will be a field of displacements represented by the matrix $u_0(n\times m)$. Note that the displacements of the r restrained points are zero. The displacement vector of the free structure is given by

$$u = u_0 \Delta T + R\beta \tag{3}$$

where $\beta(r\times 1)$ represents the amount of any of the rigid body displacements. u, like ψ, has to be orthogonal to R,

$$R^t u = R^t u_0 \Delta T + R^t R\beta = 0 \tag{4}$$

From Eqs. (1), (3), and (4), one obtains

$$\beta = -R^t u_0 \Delta T \tag{5}$$

and

$$u = (I - RR^t) u_0 \Delta T \tag{6}$$

The total displacement vector caused by the distortion and the controls will be given by

$$u_T = \psi + u \tag{7}$$

It is appropriate to minimize the Euclidian norm of u_T

$$u_T^2 = (\psi^t + u^t)(\psi + u) \tag{8}$$

Now,

$$\frac{\partial u_T^2}{\partial \Delta T} = 0 = 2u_0^t \psi + 2u_0^t [I - RR^t] u_0 \Delta T \tag{9}$$

or

$$A\Delta T = q \tag{10}$$

where

$$A = u_0^t [I - RR^t] u_0, \qquad q = -u_0^t \psi \tag{11}$$

As pointed out by Prof. Haftka,[1] in the case of force control the forces applied on the free structure must be in equilibrium. This means that any particular group of control forces can be characterized by one parameter P_i. The number of control forces is now represented by the vector $P(m\times 1)$ where m is the number of independent force parameters. Clearly, for the force control, one has to replace ΔT by P and no special treatment is needed.

Following Ref. 1, one can define an efficient coefficient

$$g^2 = u_T^2 / \psi^2 \tag{12}$$

It is easy to show that

$$g^2 = \frac{u_T^t u_T}{\psi^t \psi} = 1 - \frac{\Delta T^t q}{\psi^t \psi} \tag{13}$$

Equation (13) is identical to Eq. (16) of Ref. 1. However, the ΔT (or P) are different than those obtained by Prof. Haftka.[1] Now, g or some power of it can be minimized with respect to the placement of the control points by using some of the powerful procedures existing in the literature.[1] ΔT or P must satisfy Eq. (10). An additional constraint is that the control points must belong to the structure.

References

[1]Haftka, R. T., "Optimum Placement of Controls for Static Deformations of Space Structures," *AIAA Journal*, Vol. 22, Sept. 1984, pp. 1293-1298.

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Reply by Author to M. Baruch

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PROFESSOR Baruch's Comment covers three main points:

1) To note that in a static problem there is no need to use the mass properties (mass matrix) of the structure.

2) To present an alternate derivation of the equations for static control for discrete models.

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